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Possible dynamical determination of m_t, m_b and m_τ [1]

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Abstract

Motivated by four-dimensional superstring models, we consider the possibility of treating the Yukawa couplings of the Minimal Supersymmetric Standard Model (MSSM) as dynamical variables of the effective theory at the electroweak scale. Assuming bottom-tau unification, we concentrate on the top and bottom Yukawa couplings, and find that minimizing the effective potential drives them close to an effective infrared fixed line. Requiring an acceptable bottom-top mass ratio leads in principle to an additional constraint on the MSSM parameter space. As a by-product, we give new approximate analytical solutions of the renormalization group equations for the MSSM parameters.

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1. In a recent paper [1], the possibility was discussed of treating the Yukawa couplings of the Minimal Supersymmetric Standard Model (MSSM) not as numerical parameters but as dynamical variables (for similar suggestions, see also [2,3]). This possibility naturally arises when one embeds the MSSM into a more fundamental theory, such as supergravity or superstrings, where parameters are replaced by vacuum expectation values of some singlet scalar fields (moduli), corresponding to approximately flat directions of the effective potential. In [1], the discussion was mainly restricted to the top-quark Yukawa coupling, and it was found that, if the scale $M_{\rm SUSY}$ of the explicit MSSM mass terms is of the order of the electroweak scale, and if supersymmetry breaking does not induce mass terms larger than $\mathcal{O}({M_{\rm SUSY}}^2/M_{\rm Planck})$ in the relevant moduli directions, then minimizing the vacuum energy attracts the top-quark Yukawa coupling close to its effective infrared fixed point, which is compatible with a top-quark mass in the experimentally allowed range. In [1], it was assumed that the scale $M_{\rm SUSY}$, proportional to the gravitino mass $m_{3/2}$, also corresponds to an approximately flat direction of the fundamental theory, as suggested by a certain class of supergravity models [4,5]. However, we shall see that the result on the top-quark Yukawa coupling remains valid even if one assumes (as is often done in phenomenological analyses) that $M_{\rm SUSY}$ is fixed by some physics at very high scales, which allows $M_{\rm SUSY}$ to be treated as an input parameter, independent of the Yukawa couplings apart from the usual renormalization effects, in the effective field theory at the electroweak scale.

In the present paper, we examine the possibility of dynamically explaining the third-generation fermion masses: m_t, m_b, m_τ. To do so, we generalize the considerations of [1] to the case where all the third-generation Yukawa couplings are included, still neglecting the two light generations. Assuming for simplicity the unification relation $h_b(M_U) = h_\tau(M_U)$, which in first approximation fits the experimental value of the m_b/m_τ ratio [6,7], and treating the explicit MSSM mass terms as numerical parameters, we find that in this more complicated case the Yukawa couplings are dynamically attracted close to an effective infrared fixed line, $F(h_t, h_b) = 0$. We also find that minimization of the effective potential with respect to the residual variable θ, which parametrizes the infrared fixed line, dynamically fixes the ratio h_b/h_t, and may allow for acceptable values of the m_b/m_t ratio within the residual MSSM parameter space. Our approach leads to the elimination of two of the free parameters of the MSSM, h_t and h_b, in addition to the parameter h_τ, removed by the unification relation $h_b(M_U) = h_\tau(M_U)$. Furthermore, the infrared behaviour of the renormalization group equations (RGEs) for the mass parameters of the MSSM is such that the latter are severely constrained, with some combinations being driven close to effective infrared fixed values.

The structure of our paper is as follows. In section 2, we review the infrared behaviour of the running Yukawa couplings and masses in the MSSM [8–14], presenting some approximate analytical solutions of the RGE for the top and bottom Yukawa couplings[1], as

[1]Similar results have been simultaneously and independently obtained in [14].

well as for the MSSM mass terms, when both h_b and h_t are non-negligible. In section 3, we present the theoretical motivations that lead us to minimize the effective potential not only with respect to the Higgs fields, but also with respect to the top and bottom Yukawa couplings, taken as independent dynamical variables. In section 4 we describe in some detail the results of such a minimization, both analytically and numerically, working for given numerical boundary conditions on the MSSM mass parameters at the unification scale M_U. We begin by showing that the leading dependence of the effective MSSM potential on h_t and h_b forces the latter, upon minimization, to lie close to the effective infrared fixed line. We then proceed to the more subtle problem of minimizing the low-energy effective potential along the infrared fixed line. In section 5 we summarize our conclusions, after commenting on the case in which $M_{\rm SUSY}$ is also taken as a dynamical variable and on the case in which the top and bottom Yukawa are constrained dynamical variables.

2. Neglecting as announced the first two generations, and working as usual in a mass-independent renormalization scheme, the one-loop RGEs for the Yukawa couplings read [15]

$$\begin{aligned}
\frac{d\alpha_t}{dt} &= \frac{\alpha_t}{4\pi}\left(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{9}\alpha' - 6\alpha_t - \alpha_b\right) , \\
\frac{d\alpha_b}{dt} &= \frac{\alpha_b}{4\pi}\left(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{7}{9}\alpha' - \alpha_t - 6\alpha_b - \alpha_\tau\right) , \\
\frac{d\alpha_\tau}{dt} &= \frac{\alpha_\tau}{4\pi}\left(3\alpha_2 + 3\alpha' - 3\alpha_b - 4\alpha_\tau\right) ,
\end{aligned} \tag{1}$$

where $\alpha_{t,b,\tau} \equiv h^2_{t,b,\tau}/(4\pi)$, $t \equiv \log(M_U^2/Q^2)$ and $M_U \simeq 2 \times 10^{16}$ GeV. Two-loop RGEs are available [16], but we do not need them for our present purpose.

If in eqs. (1) we neglect α_b and α_τ with respect to α_t, which is a good first approximation when $\tan\beta \equiv v_2/v_1 \ll m_t/m_b$, and take $Q \sim M_{\rm SUSY} \sim m_Z$, then the effective infrared fixed point for the top-quark Yukawa coupling is [9,17] $\alpha_t \simeq (8/9)\alpha_3$, corresponding to a running top-quark mass $m_t \simeq (195 \text{ GeV}) \sin\beta$, from which the pole mass can be extracted by including the standard QCD corrections, $m_t^{pole} = m_t(m_t)[1+4\alpha_3(m_t)/(3\pi)+\ldots]$. When α_b and α_τ are not neglected, which is the case of interest for the present paper, eqs. (1) have a more complicated infrared behaviour. Choosing for simplicity $\alpha_b^U = \alpha_\tau^U$, in order to have a more manageable two-variable problem, the resulting infrared structure is displayed in fig. 1. We choose random boundary conditions satisfying the constraint

$$2\alpha_U < \alpha_t^U + \alpha_b^U < 1 \, , \tag{2}$$

with $\alpha_U \simeq 1/25$, corresponding to the dots in the region of the (α_t^U, α_b^U) plane shown in fig. 1a. We then solve numerically the RGEs of eq. (1) at the representative scale $Q = 200$ GeV, to obtain α_t, α_b and α_τ. The resulting region of the (h_t, h_b) plane is shown by the dots in fig. 1b. The important aspect to be stressed is the focusing effect due to the

infrared structure of the RGE: a relatively wide region of the (h_t^U, h_b^U) plane is mapped into a very thin region of the (h_t, h_b) plane, clustering around an 'effective infrared fixed line'. Another effect, clearly visible in fig. 1, is the existence of some special points along the effective infrared fixed line. If we look at the density of points in the (h_t, h_b) plane, corresponding to a uniform distribution in the (h_t^U, h_b^U) plane, we can clearly see that the point $h_t = h_b$ is an attractor, whereas $h_t = 0$ and $h_b = 0$ are repulsors.

For practical purposes, we now introduce some approximate analytical formulae for h_t and h_b, which can be useful to parametrize the effective infrared fixed line. If in the RGEs for α_t and α_b we neglect the terms proportional to α_τ and to α', and we define the auxiliary variables

$$\rho \equiv \sqrt{h_t^2 + h_b^2}\,, \qquad \tan\theta \equiv \frac{h_b}{h_t}\,, \tag{3}$$

after some calculations we can write the solution as

$$\frac{f(\sin^2 2\theta)}{\rho^2} = \frac{f(\sin^2 2\theta_U)}{\rho_U^2}\frac{1}{E} + \frac{3}{8\pi^2}\frac{F}{E}\,, \tag{4}$$

$$\frac{\rho^2(\sin 2\theta)^{12/5}}{(\cos 2\theta)^{7/5}} = \frac{\rho_U^2(\sin 2\theta_U)^{12/5}}{(\cos 2\theta_U)^{7/5}}\cdot E\,, \tag{5}$$

where

$$f(x) \equiv {}_2\mathcal{F}_1\left(1/2, 1, 11/5\,; x\right) = \frac{6}{5}\int_0^1 ds(1-s)^{1/5}(1-sx)^{-1/2}\,, \tag{6}$$

$$E(t) \equiv \left(1 - \frac{3\alpha_U}{4\pi}t\right)^{-16/9}\left(1 + \frac{\alpha_U}{4\pi}t\right)^3\,, \qquad F \equiv \int_0^t dt' E(t')\,. \tag{7}$$

The same result was independently obtained in ref. [14]. Since the behaviour of the hypergeometric function f will play an important role in the considerations of section 4, for illustration we plot in fig. 2 $f(\sin^2 2\theta)$ and $df(\sin^2 2\theta)/d\theta$, as functions of θ. From eq. (5), we can see that the renormalization of the quantity in the first member is multiplicative. On the other hand, $1 \le f(\sin^2 2\theta) \le 12/7$, and at scales $Q \sim 200$ GeV it is (still neglecting α' effects) $E \simeq 10$ and $F \simeq 220$, so that the second member of eq. (4) is dominated by $3F/(8\pi^2 E)$ for sufficiently large values of ρ_U ($\rho_U \gtrsim 0.5$). This defines an effective infrared fixed line at any scale Q close to the electroweak scale, parametrized by

$$h_t = \rho_{IR}(\theta)\cos\theta\,, \qquad h_b = \rho_{IR}(\theta)\sin\theta\,, \tag{8}$$

where

$$\rho_{IR}(\theta) \equiv \sqrt{\frac{8\pi^2 E}{3F} f(\sin^2 2\theta)}\,. \tag{9}$$

To further improve the approximation of eq. (8), we can introduce some constant shifts to fit the corrections due to α' and α_τ effects, and get

$$h_t = 0.015 + \rho_{IR}(\theta)\cos\theta\,, \qquad h_b = -0.045 + \rho_{IR}(\theta)\sin\theta\,, \tag{10}$$

where of course the range of variation of θ should be modified accordingly. It is useful to compare our formula with the one previously derived in [13], which, after correcting for α' and α_τ effects as in eq. (10), reads

$$(h_t - 0.015)^{12} + (h_b + 0.045)^{12} = \left(\frac{8\pi^2 E}{3F}\right)^6 . \tag{11}$$

Figure 1b compares the exact numerical solutions of eqs. (1), represented by the dots, with the approximate analytical solutions of eqs. (10) and (11), represented by the solid and by the dashed line, respectively. We can see that both formulae are good approximations for $h_b \ll h_t$ or $h_t \ll h_b$, whereas eq. (10) is a better approximation for $h_b \sim h_t$.

An essential ingredient in the study of the MSSM effective potential is the solutions to the RGE for the MSSM mass parameters [18]. Exact analytical solutions of the one-loop RGE are known [19] in the case of negligible h_b and h_τ. Some approximate analytical solutions have also been obtained recently for the special case $h_b = h_t$ [12]. We have improved the existing formulae by constructing approximate analytical solutions valid for any value of h_t and h_b, and including the most important α' effects, but still neglecting α_τ effects. Their explicit form is given in the Appendix. From our formulae one can easily rederive some known relations valid at special points on the effective infrared fixed curve: $A_t/m_{1/2} \simeq H_8 - H_4/2 \simeq 1.5$ and $\Delta m_2^2 \simeq (3/2)m_0^2 + (1/2)(H_2 - H_4^2/4)m_{1/2}^2 \simeq (3/2)m_0^2 + (1/2)(6.3)m_{1/2}^2$ for $\theta = 0$ [11], $A_t/m_{1/2} \simeq A_b/m_{1/2} \simeq 1.5$ and $\Delta m_1^2 \simeq \Delta m_2^2 \simeq (9/7)m_0^2 + (3/7)(6.3)m_{1/2}^2$ for $\theta = \pi/4$ [12].

3. We now present the theoretical motivations that lead us to minimizing the MSSM effective potential not only with respect to the Higgs fields, but also with respect to the top and bottom Yukawa couplings, taken as independent dynamical variables.

In a generic supergravity model, masses and couplings are field-dependent functions. This should be kept in mind when considering both the dimensionless and the dimensionful parameters of the MSSM, seen as the low-energy effective theory of an underlying supergravity model. For each given MSSM parameter, if the scalar fields that control it are frozen to their VEVs by sufficiently heavy mass terms, then such a parameter can be treated as constant, apart from standard renormalization effects, when discussing the dynamics at the electroweak scale. If, however, after integrating out the superheavy degrees of freedom, some extra singlet scalar fields are left, with no renormalizable couplings to the MSSM fields and masses $\mathcal{O}({M_{\rm SUSY}}^2/M_{\rm Planck})$ or smaller, then MSSM quantum corrections can play a role in the determination of their VEVs, and the corresponding MSSM parameters should be treated as dynamical variables of the effective theory at the electroweak scale.

Consider for example the class of supergravity models whose field content splits into an 'observable sector', containing the MSSM states (and possibly others) and a 'hidden sector', coupled to the observable sector only via interactions of gravitational strength.

The situation of interest to us can be realized in the special subclass of models [4,5] that exhibit, in their hidden sector, some approximately flat directions of the classical potential, associated to some 'moduli' fields. Such degeneracy of the classical vacuum is in general removed by quantum corrections, including the perturbative ones if supersymmetry is spontaneously broken. We would like to envisage here the possibility that the potential along some of these flat directions does not get large quantum corrections from the superheavy sectors of the theory. Then we need to minimize the effective potential at the electroweak scale to fix some moduli VEVs and to determine those MSSM low-energy parameters which carry a non-trivial dependence on such moduli.

The above possibility is supported by the general structure of four-dimensional superstring models [20], where all the low-energy parameters, in particular the Yukawa couplings, are dynamical variables depending on some moduli VEVs. Indeed, some interesting superstring solutions could give rise, in the low-energy limit, to spontaneously broken $N = 1$ effective supergravities of the type considered above. At the classical level, gauge and Yukawa couplings are related [21] by a string super-unification condition:

$$\frac{k_i}{\alpha_i} = \frac{1}{\alpha_{str}} \, . \tag{12}$$

In eq. (12), α_{str} is the coupling constant associated with the string loop expansion: already at the classical level, this is not a numerical parameter but a dynamical variable, related to the VEV of the dilaton field $(S + \bar{S})$ by $2\pi\alpha_{str} = (S + \bar{S})^{-1}$. For string solutions with unbroken supersymmetry, α_{str} is a flat direction, not only classically but at all orders in the string perturbative expansion. In the case of the gauge couplings ($i = 1, 2, 3$ for the factors of the standard model gauge group), the coefficients k_i are constants depending on the particular string solution. In the following, we shall have in mind the class of string solutions for which, with the usual normalization convention $g_1 = \sqrt{5/3}g'$, it is $k_3 = k_2 = k_1 = 1$. We shall also assume that some non-perturbative effects break spontaneously $N = 1$ supersymmetry and fix the VEV of the dilaton field, in such a way that gauge couplings are not dynamical variables at the electroweak scale. In the case of the Yukawa couplings ($i = t, b, \tau$ for the third-generation ones to be considered here), at the string classical level the coefficients k_i typically are exponentially suppressed or of order unity, as can be easily checked in many explicit examples. For instance, in free fermionic constructions the non-vanishing tree-level Yukawa couplings correspond to moduli-independent coefficients $k_i = 2$. In other classes of string solutions one can have moduli-dependent k_i coefficients for the tree-level Yukawa couplings.

Even when the tree-level moduli-dependence of the Yukawa couplings is identical to that of the gauge couplings, string-loop corrections [22] to the low-energy effective action can introduce additional, non-universal moduli dependences. One can then envisage, as already discussed in [1], various possible situations.

A first possibility is that, after the inclusion of string-loop corrections and of possible non-perturbative effects associated with supersymmetry breaking and with the stabilizitaion of the dilaton VEV, the Yukawa couplings have no residual moduli dependence. In

this case, the non-vanishing ones will still obey a superunification condition of the form (12), with $k_i = \mathcal{O}(1)$. In particular, the top and bottom Yukawa couplings will fall in the domain of attraction of the infrared fixed curve discussed in section 2.

A second possibility is that for some Yukawa couplings there is a residual moduli dependence along some approximately flat directions. In the following section, we shall assume that this moduli dependence preserves the unification relation $h_b^U = h_\tau^U$, but allows the top and bottom Yukawa couplings to be treated as independent variables of the effective theory at the electroweak scale. Of course, more complicated situations could also arise, for example that there be only one independent flat direction in moduli space. In this case, still assuming for simplicity $h_b^U = h_\tau^U$, the allowed range of variation for h_b^U and h_t^U would be restricted to a certain curve of the (h_t^U, h_b^U) plane, and this should be taken into account when minimizing the low-energy effective potential. We shall temporarily disregard this last possibility in the following section, but we shall come back to it in the concluding one.

4. If, as suggested by the models of ref. [5], there are no quantum corrections to the vacuum energy carrying positive powers of superheavy scales, the one-loop effective potential of the MSSM can be written as $V = V_0 + \Delta V$, where

$$V_0 = m_1^2 v_1^2 + m_2^2 v_2^2 + 2 m_3^2 v_1 v_2 + \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)^2 + \eta \, , \tag{13}$$

and

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-1)^{2J_i+1}(2J_i+1) m_i^4 (\log \frac{m_i^2}{Q^2} - \frac{3}{2}) \, . \tag{14}$$

In eq. (13), v_1 and v_2 are the neutral Higgs vacuum expectation values, and $m_1^2 = m_{H_1}^2 + \mu^2$, $m_2^2 = m_{H_2}^2 + \mu^2$, $m_3^2 = B\mu$ are mass parameters. The 'cosmological term' $\eta \equiv \hat{\eta} m_{3/2}^4$ takes into account, as in [1] but in a slightly different notation, the contributions to the low-energy effective potential that do not depend on the MSSM fields. Given a set of boundary conditions at M_U, the mass parameters m_1^2, m_2^2, m_3^2 have an implicit dependence on the top and bottom Yukawa couplings via their RGEs, as illustrated by the approximate analytical solutions given in the Appendix. A similar implicit dependence is present for the parameter η, whose renormalization-group evolution was studied in [1]. In eq. (14), m_i and J_i are the tree-level field-dependent mass and the spin for each particle i in the MSSM spectrum. Notice that ΔV has an explicit dependence on h_t and h_b only via the top, bottom, stop and sbottom squared masses,

$$m_t^2 = h_t^2 v_2^2 \, , \qquad m_b^2 = h_b^2 v_1^2 \, , \tag{15}$$

$$m_{\tilde{t}_{1,2}}^2 \quad = \quad h_t^2 v_2^2 + \frac{m_{Q_3}^2 + m_{U_3}^2}{2} + \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)$$

$$\pm \sqrt{\left[\frac{m_{Q_3}^2 - m_{U_3}^2}{2} + \frac{3g^2 - 5g'^2}{24}(v_1^2 - v_2^2)\right]^2 + h_t^2(A_t v_2 + \mu v_1)^2}\,, \qquad (16)$$

$$\begin{aligned} m_{\tilde{b}_{1,2}}^2 &= h_b^2 v_1^2 + \frac{m_{Q_3}^2 + m_{D_3}^2}{2} - \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2) \\ &\pm \sqrt{\left[\frac{m_{Q_3}^2 - m_{D_3}^2}{2} - \frac{3g^2 + 2g'^2}{24}(v_1^2 - v_2^2)\right]^2 + h_b^2(A_b v_1 + \mu v_2)^2}\,. \end{aligned} \qquad (17)$$

As announced, we shall discuss here the minimization of the MSSM one-loop effective potential not only with respect to the Higgs fields, v_1 and v_2, but also with respect to the top and bottom Yukawa couplings, h_t and h_b, treated as independent dynamical variables. We would like to stress once more the importance of the cosmological term η in the RG-improved tree-level potential of eq. (13). This term is usually neglected because it does not depend on v_1 and v_2; hence it does not play any significant role in the minimization with respect to the Higgs fields. In our case, however, this term must be included, since, given a boundary value $\eta_0 \equiv \eta(M_U)$, η has an implicit dependence on the Yukawa couplings via its renormalization group evolution: neglecting η would create an artificial dependence of the effective potential on the renormalization scale Q.

With the above comments in mind, we can proceed to the minimization of the one-loop effective potential with respect to the Higgs fields and the top and bottom Yukawa couplings, given a set of boundary conditions $(m_0, m_{1/2}, A_0, B_0, \mu_0\,; \eta_0)$. For convenience, we work with the polar coordinates ρ and θ already introduced in section 2, and we proceed in two separate steps. First, we fix θ to an arbitrary value, and we minimize the potential with respect to v_1, v_2 and ρ. We find that, for any given value of θ, the value of V at its minimum with respect to v_1 and v_2 gets smaller and smaller as ρ increases, until ρ reaches its maximum allowed value, $\rho_{IR}(\theta)$, corresponding to a point on the effective infrared fixed line. This result has been tested numerically for many different values of θ and of the boundary conditions on the free parameters, using the full one-loop effective potential of eqs. (13) and (14). We also verified that in most cases, with an appropriate choice of the renormalization scale, $Q^2 \sim m_{\tilde{t}_1} m_{\tilde{t}_2}$, the minimization with respect to h_t and h_b is dominated by the V_0 contribution: this extends the results obtained in [23] for the usual minimization with respect to v_1 and v_2. The mechanism of attraction towards the effective infrared fixed line can be understood semi-analytically in sufficiently simple cases, as we shall now discuss on an example.

Consider the toy version of the MSSM corresponding to $m_0 = A_0 = B_0 = \mu_0 = 0$, $v_1 = 0$, $h_b = 0$, but with $m_{1/2}$ and η_0 both taken as fixed numerical inputs and not as dynamical variables. In this case, eq. (13) simplifies to

$$V_0 = m_2^2 v_2^2 + \frac{g^2 + g'^2}{8} v_2^4 + \eta\,. \qquad (18)$$

Assuming $m_2^2 < 0$, as needed for $SU(2) \times U(1)$ breaking, and minimizing with respect to v_2, we find $V_0|_{v_2=\langle v_2^2 \rangle} = -2m_2^4/(g^2+g'^2) + \eta$, and finally

$$\left. \frac{\partial V_0}{\partial x} \right|_{v_2=\langle v_2 \rangle} = \frac{1}{2\pi} \frac{-2m_2^2}{\alpha_2+\alpha'} \frac{\partial m_2^2}{\partial x} + \frac{\partial \eta}{\partial x} \,, \tag{19}$$

where $x \equiv \alpha_t/\alpha_t^{IR} = 2\pi E \alpha_t/(3F)$. The Yukawa-coupling dependence of m_2^2 can be easily understood by specializing the general formulae given in the Appendix,

$$m_2^2 = (C + Ax + Bx^2) m_{1/2}^2 \,, \tag{20}$$

where, at scales Q of the order of the electroweak scale and in the notation of the Appendix, $C = C_1/4 + C_2 \simeq 0.5$, $A = -H_2/2 \simeq -5$, $B = H_4^2/8 \simeq 2$. We can check the well-known fact that, in the physical region $x < 1$, we always have $Ax + Bx^2 < 0$, which leads to $m_2^2 < 0$ for sufficiently large values of x. If the η-dependent part of eq. (19) can be neglected, we can already state that in the case under consideration x is driven to $x = 1$. However, we know from [1] that η (slowly) increases for increasing x, thus a quantitative comparison of the two terms in eq. (19) is necessary. From the RGE for η and α_t, we obtain

$$\frac{\partial \eta}{\partial x} = -\frac{1}{2\pi} \frac{(A+Bx)(D+Ax+Bx^2)}{G + J(1-x)} m_{1/2}^4 \,, \tag{21}$$

where, at scales Q of the order of the electroweak scale and in the notation of the Appendix, $D = 2C_3 + 2C_2 + 13C_1/18 \simeq 11.3$, $G = 16\alpha_3/3 + 3\alpha_2 + 13\alpha'/9 - 6\alpha_t^{IR} \simeq 0.01$, $J = 6\alpha_t^{IR} \simeq 0.57$. From this one can easily verify that the η-dependent part of eq. (19) is indeed negligible for all values of $x < 1$.

Having obtained the result that, for any given value of θ, minimization with respect to ρ invariably leads to $\rho = \rho_{IR}(\theta)$, we can now restrict our attention to top and bottom Yukawa couplings constrained along the effective infrared fixed line, and minimize the effective potential of the MSSM with respect to the residual angular variable θ (in addition to the usual variables v_1 and v_2). Numerical investigations show that, depending on the chosen boundary conditions for the mass parameters, different structures may appear. A typical situation is illustrated, for a representative parameter choice, in fig. 3: it corresponds to a trivial minimum for $\theta = 0$, i.e. to vanishing bottom and tau tree-level masses. As we shall discuss later, this type of structure can be rescued by some constraint on the moduli space that forbids the boundary condition $\theta_U = 0$. In this case, the low-energy θ just relaxes to its minimum allowed value, $\theta_{min} \neq 0$, and a hierarchy $0 < m_b/m_t \ll m_t$ can emerge even for values of $\tan\beta$ close to 1. In our numerical investigations, we were not able to find unconstrained non-trivial minima for $\theta \neq 0$, corresponding to universal boundary conditions on the mass parameters and a particle spectrum compatible with the present experimental data. Establishing whether realistic solutions of this kind can be obtained or not would require further investigations. If such solutions do exist, minima close to $\theta = \pi/4$ would be favoured by the peculiar behaviour of the function $f(\sin^2 2\theta)$ and by

the focusing effect of the RGEs. When $\theta \sim \pi/4$, acceptable values for m_t and m_b can be obtained only for $\tan\beta \sim m_t/m_b$. This situation can be realized either by selecting a strongly restricted region of parameter space or by allowing some violation of universality in the boundary conditions for the mass parameters.

5. Motivated by four-dimensional superstring models and their effective supergravity theories, we examined the possibility of treating the Yukawa couplings of the MSSM as dynamical variables at the electroweak scale. In particular, we concentrated on the Yukawa couplings of the third generation, neglecting the two light generations and assuming the unification relation $h_b^U = h_\tau^U$. We have found that, treating (h_t^U, h_b^U) as independent variables, minimization of the one-loop MSSM effective potential attracts (h_t, h_b) to an effective infrared fixed line. This general feature allows the elimination of one of the free parameters of the MSSM, and leads to the generic prediction

$$\frac{8}{9}\alpha_3 \lesssim \alpha_t + \alpha_b \lesssim \frac{32}{21}\alpha_3\,, \tag{22}$$

which can be further improved by including α_τ, α', higher-loop and threshold corrections. In terms of the top and bottom quark running masses, the above prediction reads

$$(M_t^{IR})^2 \lesssim \frac{m_t^2}{\sin^2\beta} + \frac{m_b^2}{\cos^2\beta} \lesssim \frac{12}{7}(M_t^{IR})^2\,, \tag{23}$$

where $M_t^{IR} \simeq (4/3)\sqrt{\alpha_3/(\alpha_2+\alpha')}m_Z \simeq 195$ GeV. This result can be translated into a relation involving the pole top and bottom masses by straightforward inclusion of some finite MSSM one-loop corrections, dominated by standard QCD effects. The ratio α_b/α_t is also determined by minimization, but its actual value at the minimum depends on the free mass parameters of the MSSM $(m_0, m_{1/2}, A_0, B_0, \mu_0\,; \eta_0)$. The number of the MSSM free parameters is further reduced by one, but no generic prediction can be made in the absence of a theory of the mass parameters.

Our results were obtained under two important assumptions. First, the overall scale $M_{\rm SUSY}$ of the MSSM mass parameters, proportional to the gravitino mass, was not taken as a dynamical variable but as a fixed numerical input. Second, the two Yukawa couplings h_t^U and $h_b^U = h_\tau^U$ were considered as independent variables in the minimization. We would like to conclude our paper by commenting on the effects of relaxing each of these two assumptions.

When, as in [1], $M_{\rm SUSY}$ is also considered as a dynamical variable, one obtains an additional constraint on the MSSM mass parameters, coming from the minimization condition with respect to the gravitino mass, which sets the overall MSSM mass scale,

$$m_{3/2}^2 \frac{\partial V_1}{\partial m_{3/2}^2} = 2V_1 + \frac{{\rm Str}\,\mathcal{M}^4}{64\pi^2} = 0\,. \tag{24}$$

It was shown in [1] that the above equation allows for the dynamical generation of the desired $M_{\rm SUSY}/M_{\rm Planck}$ hierarchy in a large region of the parameter space. The main quantitative result on the dynamical determination of the Yukawa couplings, i.e. the generic attraction of $\alpha_t + \alpha_b$ towards the effective infrared fixed line, remains the same. However, the determination of the ratio α_b/α_t as a function of the boundary conditions for the residual free mass parameters would require a separate analysis, since the minimization condition with respect to $m_{3/2}$, eq. (24), induces a non-trivial dependence of the overall mass scale of the effective potential on the angular variable θ parametrizing the effective infrared fixed line: this might allow for an easier generation of phenomenologically acceptable non-trivial minima at $\theta \neq 0$.

Another possibility is that the top and bottom Yukawa couplings at the unification scale, h_b^U and h_t^U, are not independent but constrained by some functional relation, corresponding to a curve in the (h_t^U, h_b^U) plane, such as those shown in fig. 1c. Such a possibility can occur if the moduli dependences of h_b^U and h_t^U are correlated, and correspond to a single independent flat direction in moduli space: in this case also the number of independent minimization conditions has to be restricted accordingly. However, the generic phenomenon of attraction towards the effective infrared fixed line will persist also in this case, as long as the constraint on (h_t^U, h_b^U) allows for some points with sufficiently large ρ_U to fall in its domain of attraction. If the constraint at the unification scale allows for all possible values of θ_U within the domain of attraction of the effective infrared fixed line, then such a curve is mapped into the entire infrared fixed line, and, as far as low-energy Yukawa couplings are concerned, minimization under the constraint gives exactly the same result as unconstrained minimization. It might well be, however, that either the range of variation of θ_U is restricted, or a sufficently large value of ρ_U is allowed only for certain values of θ_U, as is the case for the dot-dashed and solid curves in fig. 1c, respectively. In this case the minima of the low-energy potential will still lie on the infrared fixed line, but minimization with respect to θ must take into account the bounds set by the constraint at the unification scale, as apparent from the corresponding curves in fig. 1d. The only case in which the constrained minimum does not lie along the effective infrared fixed line corresponds to a curve in the (h_t^U, h_b^U) plane that does not allow for sufficiently large values of ρ_U: we regard this last situation, exemplified by the dashed lines in figs. 1c and 1d, as extremely unlikely, given the fact that the tree-level string Yukawa couplings are typically of the order of the unified gauge coupling, which falls already in the domain of attraction of the effective infrared fixed line. Several possible constraints on the Yukawa couplings at the unification scale were recently conjectured in [3], but without dwelling into a possible string origin. We have argued here that the detailed form of these constraints may or may not be relevant for the determination of the low-energy Yukawa couplings. The present understanding of the moduli space of four-dimensional superstring models, with its generalized duality symmetries, indeed suggests the possible existence of such a constraint, but does not allow to single out a specific form for it in a model-independent way.

Acknowledgements

We thank M. Carena and C.E.M. Wagner for many useful conversations.

Appendix

We present here some approximate analytical solutions to the RGE for the MSSM mass parameters, which include h_t and h_b effects for any values of the latter, but still neglect h_τ effects. We assume for simplicity universal boundary conditions at the unification scale:

$$M_3(M_U) = M_2(M_U) = M_1(M_U) \equiv m_{1/2}, \tag{25}$$

$$\begin{aligned} \tilde{m}^2_{Q_a}(M_U) &= \tilde{m}^2_{U^c_a}(M_U) = \tilde{m}^2_{D^c_a}(M_U) = \tilde{m}^2_{L_a}(M_U) \\ &= \tilde{m}^2_{E^c_a}(M_U) = m^2_{H_1}(M_U) = m^2_{H_2}(M_U) \equiv m_0^2\,, \end{aligned} \tag{26}$$

$$A^U(M_U) = A^D(M_U) = A^E(M_U) \equiv A_0\,, \quad B(M_U) \equiv B_0\,, \quad \mu(M_U) \equiv \mu_0\,. \tag{27}$$

To parametrize the dependence of the relevant mass parameters on the top and bottom Yukawa couplings, we introduce the auxiliary variables

$$x \equiv \left(\frac{h_t}{\sqrt{\frac{8\pi^2 E}{3F}}} \right)^2 , \qquad y \equiv \left(\frac{h_b}{\sqrt{\frac{8\pi^2 E}{3F}}} \right)^2 . \tag{28}$$

On the infrared curve of eq. (8), we can write $x = f(\sin^2 2\theta)\cos^2\theta$ and $y = f(\sin^2 2\theta)\sin^2\theta$. To include the most important α' effects, in such a way that our approximate formulae are optimized for $x \gtrsim y$, we define

$$E \equiv Z_3^{\frac{16}{9}} Z_2^{-3} Z_1^{-\frac{13}{99}}\,, \qquad F \equiv \int_0^t E(t')dt'\,, \tag{29}$$

where $(i = 1,2,3)$

$$Z_i \equiv \left(1 + \frac{b_i}{4\pi} t\right)^{-1}\,, \tag{30}$$

and

$$b_3 = -3\,, \quad b_2 = 1\,, \quad b_1 = \frac{33}{5}\,. \tag{31}$$

The low-energy mass parameters with a non-trivial dependence on x and y can be written as

$$m^2_{Q_3} = m_0^2 + (\frac{1}{36}C_1 + C_2 + C_3)m^2_{1/2} - \frac{1}{3}\left(\Delta m_1^2 + \Delta m_2^2\right)\,, \tag{32}$$

$$m^2_{U_3} = m_0^2 + (\frac{4}{9}C_1 + C_3)m^2_{1/2} - \frac{2}{3}\Delta m_2^2\,, \tag{33}$$

$$m^2_{D_3} = m_0^2 + (\frac{1}{9}C_1 + C_3)m^2_{1/2} - \frac{2}{3}\Delta m_1^2\,, \tag{34}$$

$$m^2_{H_1} = m_0^2 + (\frac{1}{4}C_1 + C_2)m^2_{1/2} - \Delta m_1^2\,, \tag{35}$$

$$m^2_{H_2} = m_0^2 + (\frac{1}{4}C_1 + C_2)m^2_{1/2} - \Delta m_2^2\,, \tag{36}$$

$$\mu^2 = \mu_0^2 \left(\frac{\alpha_t}{\alpha_t^U}\frac{\alpha_b}{\alpha_b^U}\right)^{3/7} Z_3^{-32/21} Z_2^{-3/7} Z_1^{-1/231}\,, \tag{37}$$

$$B = B_0 - \frac{1}{2}A_0(x+y) + m_{1/2}[H_9 - \frac{1}{4}H_4(x+y)]\,, \tag{38}$$

$$A_t = A_0(1 - x - \frac{1}{6}y) + m_{1/2}[H_8 - \frac{1}{2}H_4(x + \frac{1}{6}y)]\,, \tag{39}$$

$$A_b = A_0(1 - \frac{1}{6}x - y) + m_{1/2}[\tilde{H}_8 - \frac{1}{2}H_4(\frac{1}{6}x + y)]\,, \tag{40}$$

$$A_\tau = A_0(1 - \frac{1}{2}y) + m_{1/2}(H_{10} - \frac{1}{4}H_4 y)\,, \tag{41}$$

where

$$C_1 \equiv \frac{2}{11}(1 - Z_1^2)\,, \qquad C_2 \equiv \frac{3}{2}(1 - Z_2^2)\,, \qquad C_3 \equiv -\frac{8}{9}(1 - Z_3^2)\,. \tag{42}$$

The only two independent quantities entering the solutions for the soft scalar masses are

$$\Delta m_1^2 = \frac{3}{2}m_0^2 y + \frac{1}{2}A_0 y[1 - a(x,y)y](H_4 m_{1/2} + A_0) + \frac{1}{2}m_{1/2}^2 y[\tilde{H}_2 - \frac{1}{4}\; a(x,y)H_4^2 y]\,, \tag{43}$$

$$\Delta m_2^2 = \frac{3}{2}m_0^2 x + \frac{1}{2}A_0 x[1 - a(x,y)x](H_4 m_{1/2} + A_0) + \frac{1}{2}m_{1/2}^2 x[H_2 - \frac{1}{4}\; a(x,y)H_4^2 x]\,, \tag{44}$$

where

$$a(x,y) = \frac{7f[\frac{4xy}{(x+y)^2}] + 23}{30} \tag{45}$$

is a suitable interpolating function, such that $a(x,0) = a(0,y) = 1$, $a(x, y = x) = 7/6$, and

$$H_2 \equiv \frac{E}{F}tH_8\,, \qquad \tilde{H}_2 \equiv \frac{E}{F}t\tilde{H}_8\,, \qquad H_4 \equiv 2\left(t\frac{E}{F} - 1\right)\,, \tag{46}$$

$$H_8 \equiv \frac{\alpha_U}{4\pi}t\left(\frac{16}{3}Z_3 + 3Z_2 + \frac{13}{15}Z_1\right)\,, \qquad H_9 \equiv \frac{\alpha_U}{4\pi}t(3Z_2 + \frac{3}{5}Z_1)\,, \tag{47}$$

$$\tilde{H}_8 \equiv \frac{\alpha_U}{4\pi}t\left(\frac{16}{3}Z_3 + 3Z_2 + \frac{7}{15}Z_1\right)\,, \qquad H_{10} \equiv \frac{\alpha_U}{4\pi}t\left(3Z_2 + \frac{9}{5}Z_1\right)\,. \tag{48}$$

The above formulae have been tested numerically by comparing them with the exact numerical solutions of the one-loop RGE. If one compares with exact numerical solutions neglecting α_τ effects, our approximate results are correct with less than 3% error. When α_τ effects are included in the comparison, the error of our formulae grows up to a maximum of 10%.

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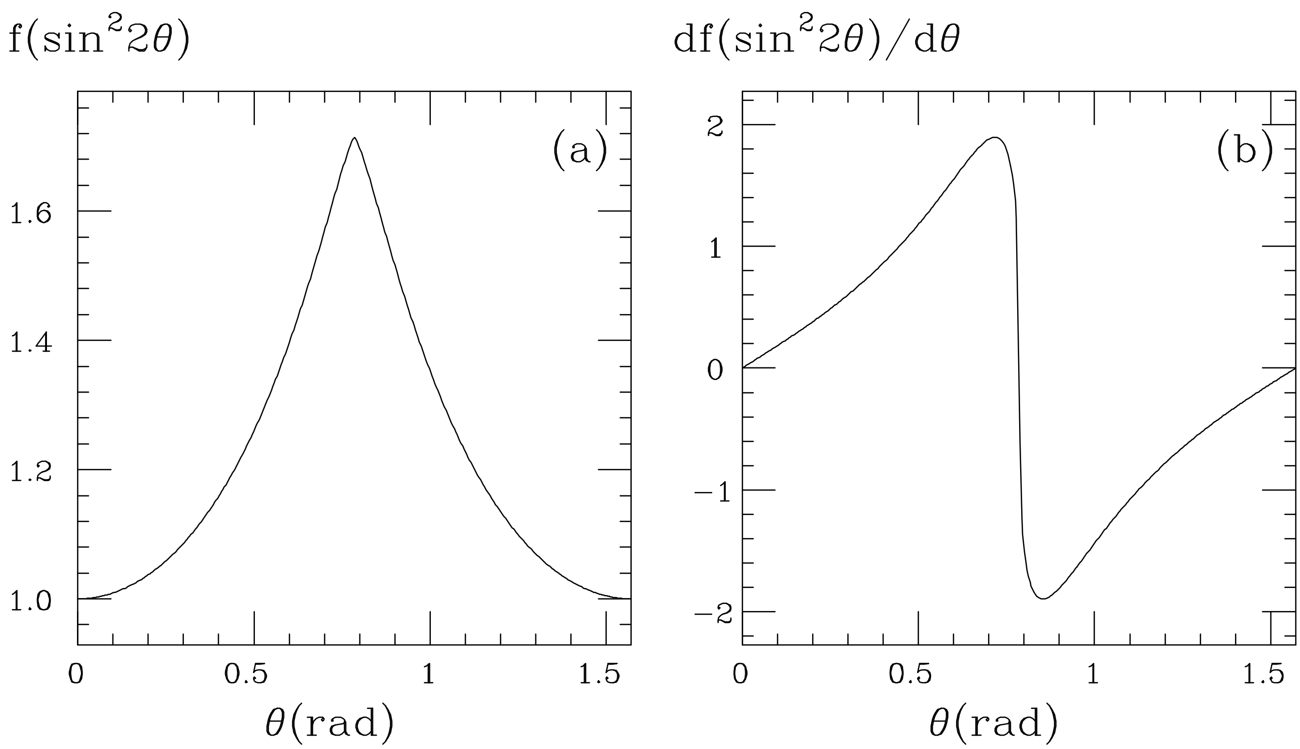
Figure captions

Fig.1: Mapping of the (h_t^U, h_b^U) plane into the (h_t, h_b) plane, for $Q = 200$ GeV, $h_b^U = h_\tau^U$, $M_{\rm U} = 2 \times 10^{16}$ GeV, $\alpha_U = 1/25$. In (b), the dots correspond to the exact numerical solutions of the one-loop RGE of eqs. (1), for the boundary conditions given in (a); the solid line corresponds to the approximate analytical solution of eq. (10), and the dashed line to the approximate solution of eq. (11). In (c) and (d), we show how some possible constraints, corresponding to curves in the (h_t^U, h_b^U) plane, are mapped into corresponding curves in the (h_t, h_b) plane.

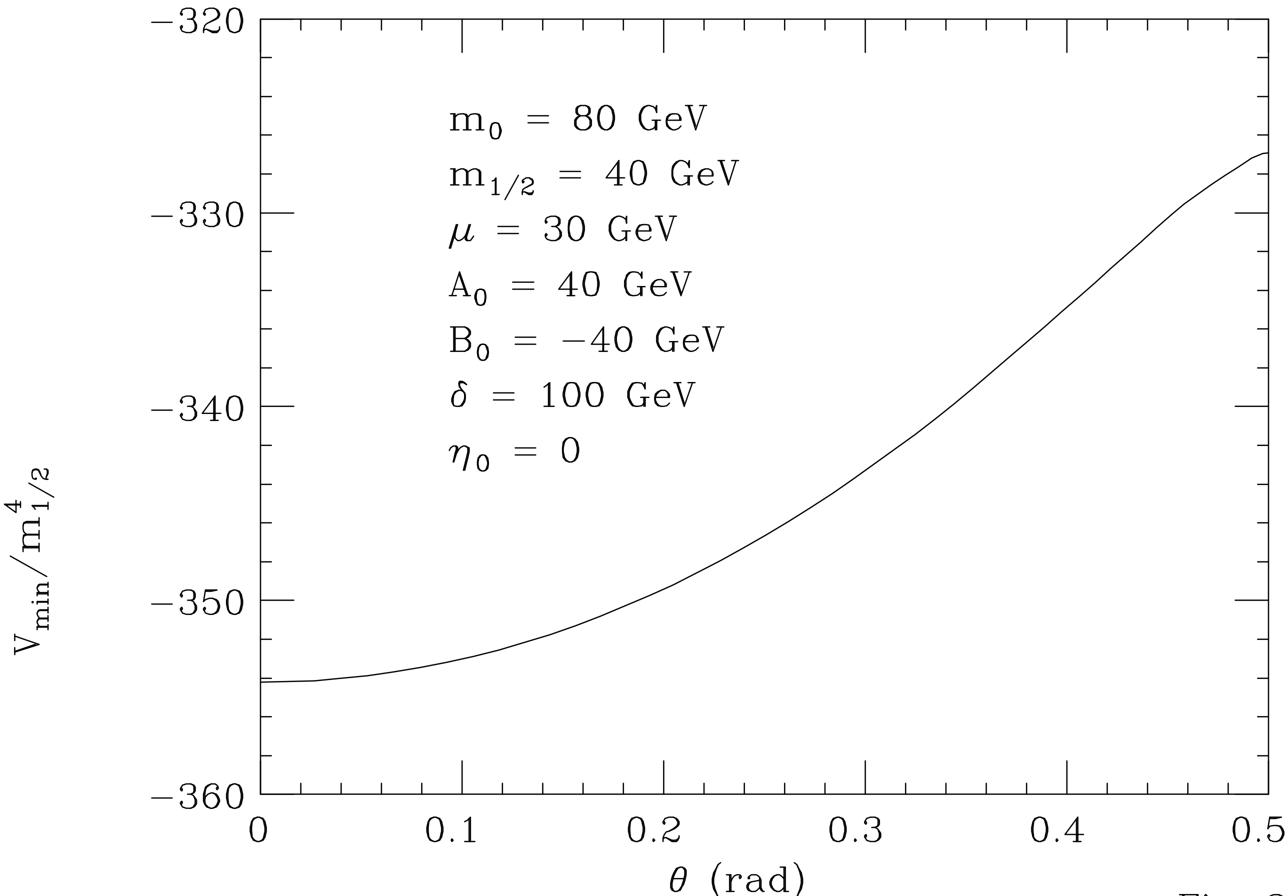
Fig.2: The function $f(\sin^2 2\theta)$ and its derivative $df(\sin^2 2\theta)/d\theta$.

Fig.3: $V(\theta)$ for a representative choice of the boundary conditions $(m_0, m_{1/2}, A_0, B_0, \mu_0)$ and for $\eta_0 = 0$. For convenience, a non-universal contribution δ to the boundary condition on m_{H_1} has been allowed: $m_{H_1}^2(M_{\rm U}) = m_0^2 + \delta^2$.



f(sin²2θ)
(a)
1.6
1.4
1.2
1.0
0
0.5
1
1.5
θ(rad)
df(sin²2θ)/dθ
(b)
2
1
0
−1
−2
0
0.5
1
1.5
θ(rad)

Fig. 2



m_0 = 80 GeV
$m_{1/2}$ = 40 GeV
μ = 30 GeV
A_0 = 40 GeV
B_0 = −40 GeV
δ = 100 GeV
η_0 = 0
−320
−330
−340
−350
−360
0
0.1
0.2
0.3
0.4
0.5
$V_{min}/m_{1/2}^4$
θ (rad)

Fig. 3

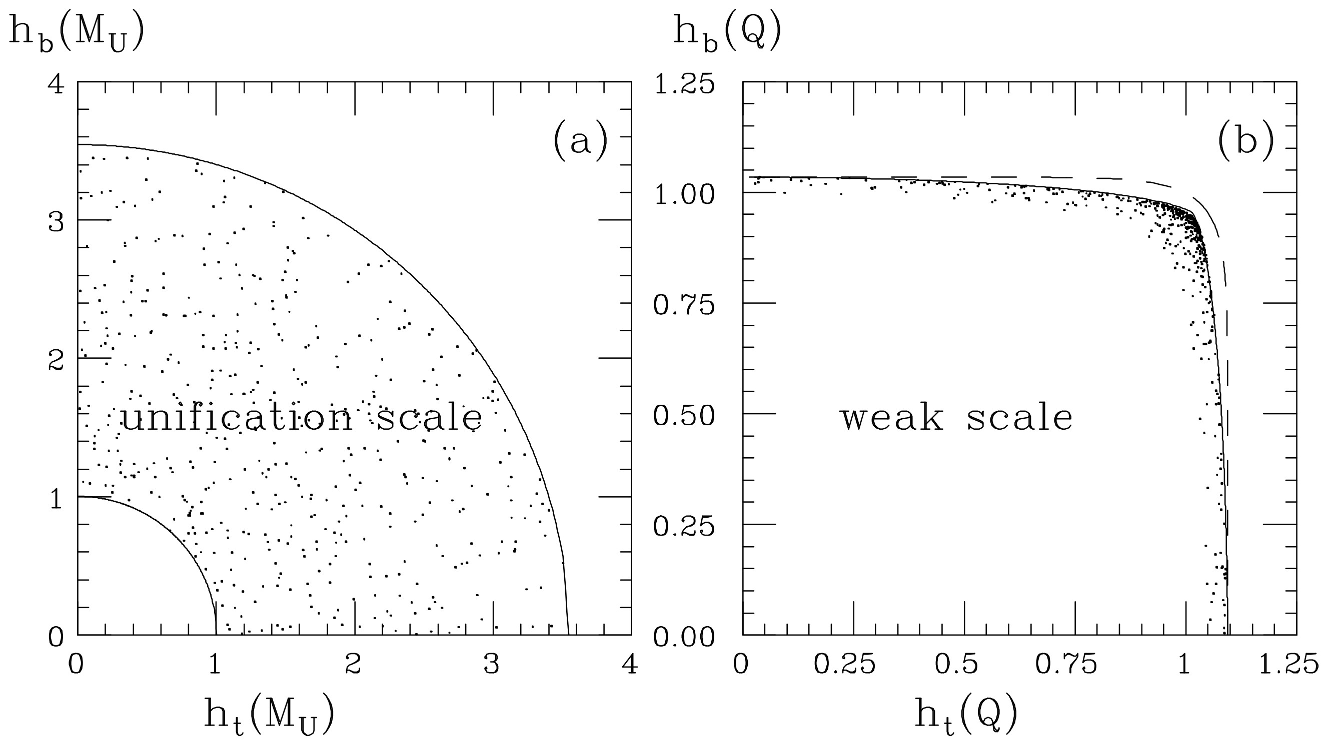
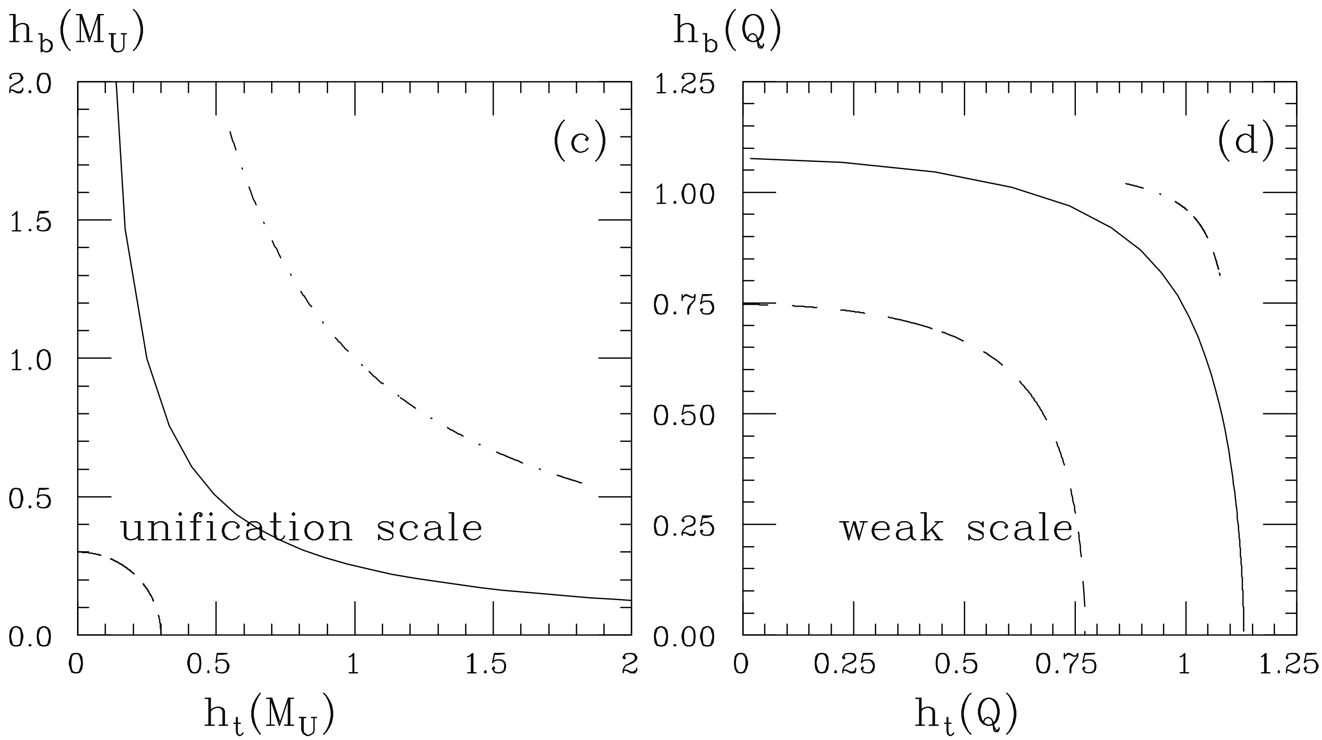


Fig. 1



$h_b(M_U)$
2.0
1.5
1.0
0.5
0.0
(c)
unification scale
0
0.5
1
1.5
2
$h_t(M_U)$
$h_b(Q)$
1.25
1.00
0.75
0.50
0.25
0.00
(d)
weak scale
0
0.25
0.5
0.75
1
1.25
$h_t(Q)$

Fig. 1